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A new method to determine a fractal dimension of non-stationary biological time-serial data

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Abstract

We devised a new analysis using quartile deviation of integrated and subtracted fluctuation, termed QIS-A, to determine a fractal dimension of non-stationary fluctuation. In the algorithm, computations of the quartile deviation, Q(n), of all integrated and subtracted fluctuations are repeated over all scales (n). The fractal scaling exponent is determined as a slope of the line relating log Q(n) to log n. Comparison of the QIS-A and a spectral analysis using 20 computer-simulated fractional Brownian motions demonstrates robustness of the QIS-A to non-stationary fluctuations. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Non-linear analysis; Fractal dimension; Non-stationary fluctuation

1. Introduction

The fractal structure of time series in biological processes has received much attention since Mandelbrot published his book *The Fractal Geometry of Nature* in 1982 [1]. Self-similarity which is equal to fractal structures of time series has been reported in the temporal fluctuations of many processes including heart rate [2,3], blood pressure [4], and fetal breathing [5,6]. Self-similarity in these time series is different from a pattern driven by uncorrelated stimuli with

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characteristic time scales (i.e., frequency related to the stimuli). In other words, these fluctuations which are satisfied with the power law scaling relationship may have different correlation properties raised from complex nonlinear dynamical systems rather than those raised from environmental stimuli [7]. Therefore, many researchers have tried to calculate the fractal dimension of biological time series to understand a generating mechanism as well as to measure a correlation [8].

Previously applied statistical measures for fractal dimension (e.g., spectral analysis and Hust rescaled range analysis), however, have been highly sensitive to non-stationarity that is a characteristic in biological time series [9]. For example, an immediate problem facing researchers applying spectral analysis to interbeat interval data is that heart beat time series are highly non-stationary, i.e., the presence of patchy patterns that change over time [10]. In the present study, we first introduced a new analysis using quartile deviation of integrated and subtracted fluctuations, termed QIS-A, to determine a fractal dimension of non-stationary fluctuation in Section 2. Second, the QIS-A was compared with a spectral analysis, using computer-simulated fractional Brownian motions (FBMs).

2. Methods

2.1. QIS-A algorithm

The QIS-A algorithm is composed of two processes: four steps for the property of fluctuation and repetition of the four steps for the fractal scaling exponent. These two processes are illustrated in Figs. 1 and 2, which show an example in the case of a computer-simulated FBM with $D = 1.9$, $N = 2400$, and $n = 160$, where D, N, and n indicate fractal dimension, total number of data, and number in each box, respectively.

In the four steps of the QIS-A, the N-point fluctuation, $F(i)$, is first divided into boxes of equal length, n: $D_k(j) = F(nk + j)$, where $F(nk + j)$ is the j-th data of the k-th box in the fluctuation. Second, the deviations of a divided fluctuation from the local mean is integrated in each box: $G_k(j) = \Sigma_{m=1}^{j}[D_k(m) - \text{mean}D_k]$, where $D_k(m)$ and meanD_k are the m-th data and the average of $D_k(m)$ in the k-th box, respectively. Third, the mean of $G_k(j)$ was subtracted from $G_k(j)$ in the k-th box: $S_k(j) = G_k(j) - \text{mean}G_k$, where $G_k(j)$ and meanG_k are the j-th data and the average of $G_k(j)$ in the k-th box, respectively. Fourth, the quartile deviation of these integrated and subtracted fluctuations in all boxes, i.e., $S_k(j)$, denoted by Q(n), is calculated as a half of range from 25% to 75% point of the cumulative curve.

These 4-step computations are repeated over scales of box size (n) ranging from 4 to $N/10$ with a step of about log 2. In order to reveal a relationship between Q(n) and the box size n which is the size of the window of observation, the fluctuation is characterized by a scaling exponent α, the slope of the line relating log Q(n) to log n which is calculated by the least squares method. A linear relationship on a double log graph indicates the presence of fractal scaling.

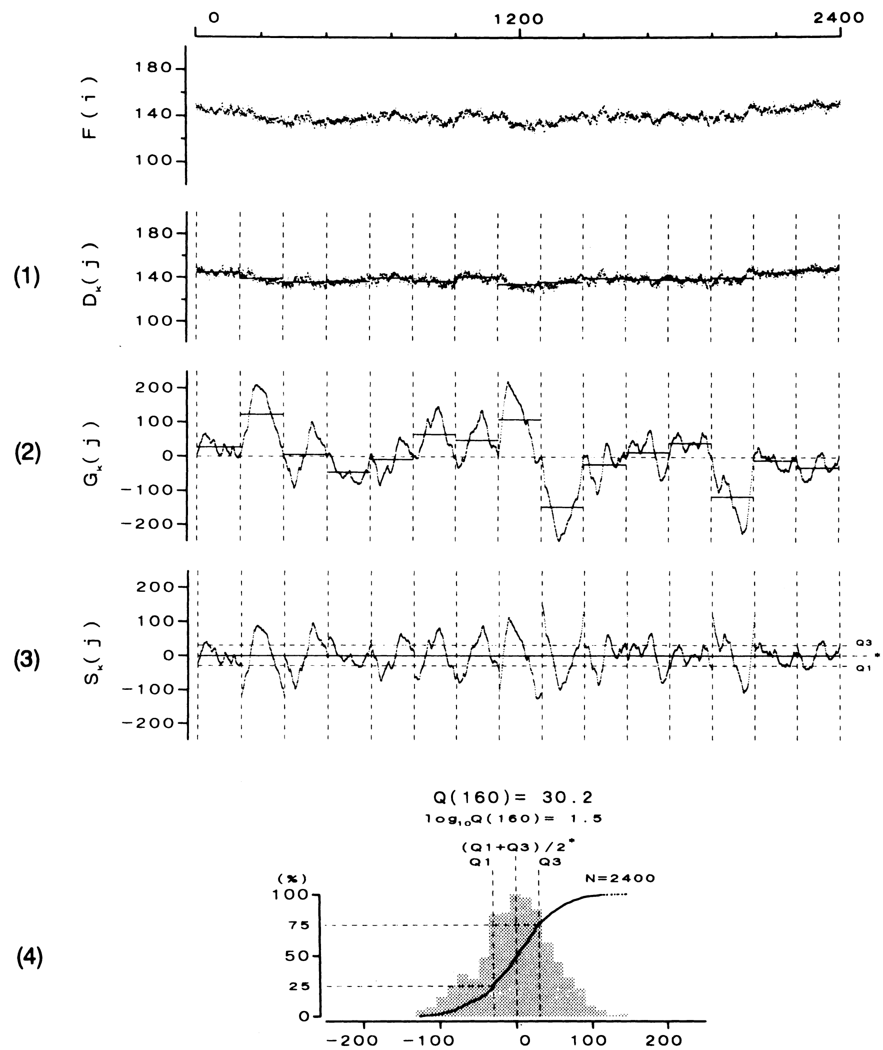

Fig. 1. An original fluctuation and example of four steps for property of variation in the QIS-A algorithm when a data and box lengths are 2400 and 160, respectively. (a) The original fluctuation with $D = 1.9$, where D indicates a fractal dimension: $F(i)$, $1 \leqq i \leqq 2400$. (b) The value of the j-th point in the k-th box of equal length, 160 points, in the 2400-point fluctuation, $F(i)$, and the mean in the k-th box: $D_k(j) = F(160k + j)$ and meanD_k, respectively. (c) The integrated fluctuation and the mean in the k-th box: $G_k(j) = \Sigma_{m=1}^{j}[D_k(m) - \text{mean}D_k]$ and meanG_k, respectively. (d) The subtracted fluctuation in the k-th box: $S_k(j) = G_k(j) - \text{mean}G_k$. (e) The histogram and the quartile deviation (Q(160)) of $S_k(j)$: Q(160) which is equal to a half of range from 25% (Q1) point to 75% (Q3) point of the cumulative curve of $S_k(j)$.

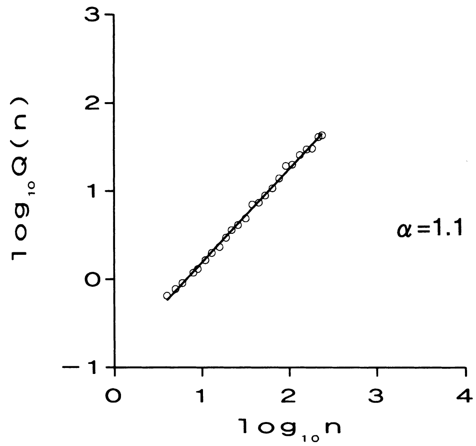

Fig. 2. Example of determination of a scaling exponent α in the QIS-A after repetition of four steps for property of variation in Fig. 1. The scaling exponent α, i.e., the slope of the line relating log Q(n) to log n, was calculated by the least squares method. The best-fit line has a slope of 1.1: $\alpha = 1.1$.

2.2. QIS-A vs spectral analysis

2.2.1. Spectral analysis

For spectral analysis, the Blackman–Tukey method [11] was applied to computer-simulated FBMs consisting of 2400 points after pretreatments of the original fluctuations, such as normalization by mean and standard deviation, detrending [2], and smoothing by Hanning window. Power spectral densities (PSDs) were estimated over box sizes ranging from 4 to 240 points/box, i.e., time scales ranging from 4 to 240 points/cycle, and were illustrated on a log–log plane. A scaling exponent β, the slope of PSDs ($-\beta$), was calculated by the least squares method, as illustrated in Fig. 3.

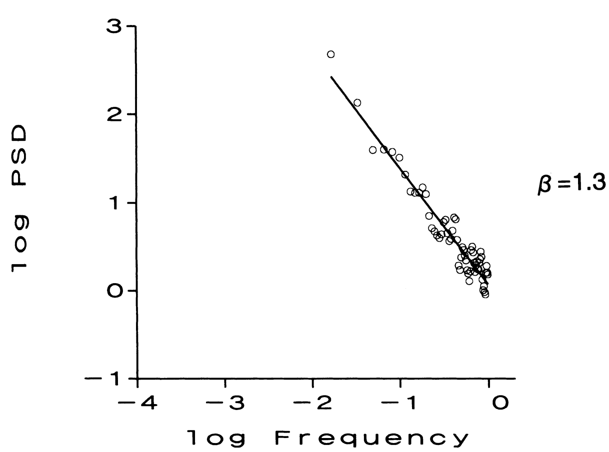

Fig. 3. Power spectral densities of the fluctuation shown in Fig. 1 on log–log plane. The best-fit line has a slope of −1.3: $\beta = 1.3$.

2.2.2. Analyses of computer-simulated FBMs

The FBM model can be understood as a generalization of Brownian motion, since Mandelbrot and Van Ness defined FBM as a moving average of the increments of Brownian motion [12]. Using this model, we made 20 computer-simulated FBMs composed of 2400 points with mean (±SD) of 140 (±5) in each D ranging from 1.0 to 2.5 with a step of 0.05. After analyses of these fluctuations by the QIS-A and the spectral analysis, the relationships between FSEs (mean α and mean β) and D were examined.

3. Results

The relationship between mean α and D (Fig. 4) was different from that between mean β and D (Fig. 5): Mean α (±SD) and mean β (±SD) in each D, which were calculated by the QIS-A and the spectral analysis, were shown in Table 1. The relationship between mean α and D was completely linear in the D range of 1.2–2.5: 14 of 16 mean αs ($1.2 \leqq D \leqq 2.5$) lay on $\alpha = 3.0 - D$. Two remaining mean αs ($D = 1.0$ and $D = 1.1$) showed the difference of 0.1 from the line of the above-described equation. In the latter relationship, only six of 16 mean βs ($2.0 \leqq D \leqq 2.5$) lay on the theoretical line ($\beta = 5.0 – 2.0D$) [13]. The differences between mean βs and this theoretical line gradually increased from 0.1 to 0.2 with the increase in non-stationarity from $D = 1.9$ to $D = 1.4$, and rapidly increased from 0.1 to 0.6 with the increase in non-stationarity from $D = 1.3$ to $D = 1.0$. Furthermore, the SD of a scaling exponent α in each D was 0.0 except for $D = 1.1$, while the SD of a scaling exponent β in each D was different from 0.0 except for $D = 2.3$ and $D = 2.4$. Thus the SDs of a scaling exponent α were less than those of a scaling exponent β in the non-stationary range ($D < 2.0$).

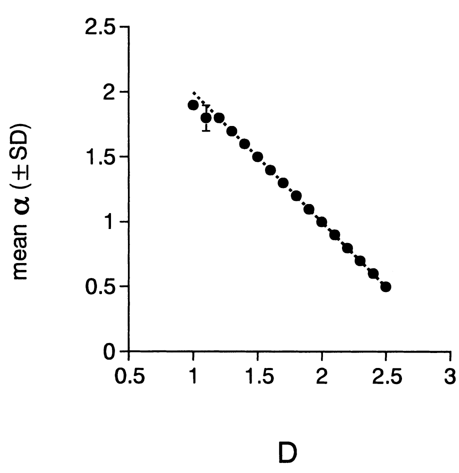

Fig. 4. The relationship between a scaling exponent α (mean ± SD) and D (shown in Table 1). Fourteen of 16 mean αs (●) lie on the dotted line, $\alpha = 3.0 - D$.

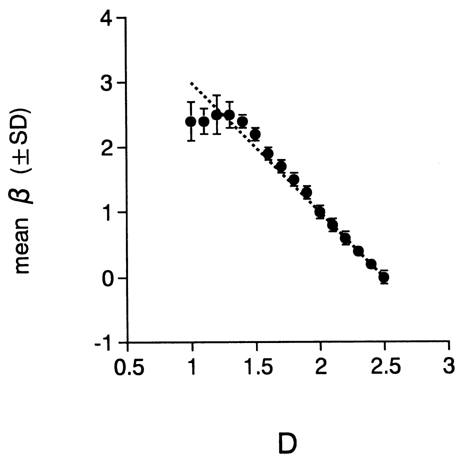

Fig. 5. The relationship between β (mean ± SD) and D (shown in Table 1). Six of 16 mean βs (●) lie on the dotted line, $\beta = 5.0 - 2.0D$.

4. Discussion

The superiority of the QIS-A to a representative conventional method, spectral analysis, was revealed by the following two results of the present study. First, the relationship between mean α and D in the non-stationary range ($D < 2$) was linear except for $D = 1.0$ and $D = 1.1$, while

Table 1
Mean α (±SD) and mean β (±SD) of 20 computer-simulated fractional Brownian motions in each fractal dimension (D)[a]

D	α mean (±SD)	β mean (±SD)
1.0	1.9 (±0.0)	2.4 (±0.3)
1.1	1.8 (±0.0)	2.4 (±0.2)
1.2	1.8 (±0.0)	2.5 (±0.3)
1.3	1.7 (±0.0)	2.5 (±0.2)
1.4	1.6 (±0.0)	2.4 (±0.1)
1.5	1.5 (±0.0)	2.2 (±0.1)
1.6	1.4 (±0.0)	1.9 (±0.1)
1.7	1.3 (±0.0)	1.7 (±0.1)
1.8	1.2 (±0.0)	1.5 (±0.1)
1.9	1.1 (±0.0)	1.3 (±0.1)
2.0	1.0 (±0.0)	1.0 (±0.1)
2.1	0.9 (±0.0)	0.8 (±0.1)
2.2	0.8 (±0.0)	0.6 (±0.1)
2.3	0.7 (±0.0)	0.4 (±0.0)
2.4	0.7 (±0.0)	0.2 (±0.0)
2.5	0.5 (±0.0)	0.0 (±0.1)

[a] The fractal scaling exponents, α and β, are calculated by the QIS-A and the spectral analysis, respectively.

the relationship between mean β and D in the same range was not linear because of high sensitivity to non-stationarity [2] and interaction of cyclic components with the residual FBM [14]. Second, the SD of a scaling exponent β in each D was larger than or equal to that of a scaling exponent α because of instability of spectral analysis. Leakage and estimated error of the spectrum at a low frequency range in spite of the pretreatments of the original fluctuations, as described in Section 2, might disturb the precise calculation of fractal dimension as well [11]. In the QIS-A, we speculate that strong linearity between a scaling exponent α and D would be guaranteed by the second, third and fourth step of the QIS-A algorithm in Fig. 1: the integration process, the subtraction process, and calculation of a quartile deviation, respectively, as described in Section 2. These steps also reveal the difference between the QIS-A and other methods.

Concerning the second step of the QIS-A, Hurst, who spent a life studying the problems related to water storage of the Nile river, adopted integration of original data in the rescaled range analysis for the first time in 1965 [15]. Our adoption of integration in the QIS-A algorithm is not only based on Feder's computer simulation of Hurst's empirical law, i.e., the rescaled range analysis [16], but practically given a lot of direction by the detrended fluctuation analysis (DFA) devised by Peng et al. [17] although the DFA is different from the QIS-A in a period of integration and the steps after integration. Peng et al. also demonstrated the merits of integration of original non-stationary fluctuation in the analyses of highly heterogeneous DNA sequences [18] and other complex physiological signals [9,19].

In the third step, the subtraction process might have the same significance as the elimination of linear trends in the pretreatment of spectral analysis [2] as well as in the DFA [17]. In the fourth step, many properties of original and integrated fluctuations to determine a fractal dimension have been proposed as follows: mean [20], relative dispersion (standard deviation/mean) [21], Fano factor (variance/mean) [22], root mean square [18], and rescaled range (difference between the maximum and minimum/standard deviation) [6]. From a statistical point of view, the quartile deviation adopted in the QIS-A could be a more suitable property of variation than the other properties, especially standard deviation [21], variance [22] and root mean square [18], because the distributions of properties were not always the Gaussian distribution.

Taking the linear relationship between mean α and D (Fig. 4) into consideration, any time-series can be characterized and understood by a scaling exponent α [9,14]. A process where the value at one point is completely uncorrelated to any previous values ($D = 2.5$), i.e., white noise, shows $\alpha = 0.5$. When a time series has statistical stationarity ($2.0 < D < 2.5$), the range of a scaling exponent α is greater than 0.5 and less than 1.0. In this range, a scaling exponent α indicates power-law correlation such that large and small values are more likely to alternate [9]. In a non-stationary range ($1.0 < D < 2.0$), a scaling exponent α ranging greater than 1.0 and less than 1.5 ($1.5 < D < 2.0$) indicates that past increase and decrease of time series are more likely to alternate in future change [14]. In contrast, a scaling exponent α ranging from greater than 1.5 and less than or equal to 2.0 ($1.0 \leqq D < 1.5$) indicates a different type of power-law correlation such that past increase is more likely to be followed by a future increase and vice versa [14]. Special cases of $\alpha = 1.0$ and $\alpha = 1.5$ correspond to $1/f$ noise and Brownian noise, respectively. The former implies the border between stationarity and non-stationarity, and the latter is made by the integration of white noise. In addition, a scaling exponent α can be viewed as an indicator that describes the "roughness" of fluctuation as well.

There are a lot of non-stationary biological processes in nature [1]. Our study shows that the QIS-A will be used as a tool for non-linear analysis of any biological non-stationary fluctuations, such as processes in time and one-dimensional objects in space.

5. Summary

We devised a new analysis using quartile deviation of integrated and subtracted fluctuation, termed QIS-A, to determine a fractal dimension of non-stationary fluctuation. In the algorithm, a fluctuation is divided into boxes of equal length (n), and deviations from the local mean in each box are integrated. This integrated fluctuation is shifted by subtraction of its own local mean. The quartile deviation, Q(n), of all integrated and subtracted fluctuations is calculated. These computations are repeated over all scales (n). The fractal scaling exponent (FSE) is determined as a slope (α) of the line relating log Q(n) to log n. In order to compare the QIS-A with a spectral analysis whose FSE was a slope ($-\beta$) of power spectral densities on log–log plane, mean α and mean β ($\pm$SDs) of 20 computer-simulated fractional Brownian motions in each D ($1.0 \leqq D \leqq 2.5$) were calculated. In the results, 14 of 16 mean αs satisfied $\alpha = 3.0 - D$, while only 6 of 16 mean βs satisfied the theoretical equation; $\beta = 5.0 - 2.0D$. The SDs of α were smaller. These results demonstrate robustness of the QIS-A to non-stationary fluctuations.

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