

# Emergence of complex dynamics in a simple model of signaling networks

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Edited by Nancy J. Kopell, Boston University, Boston, MA, and approved September 9, 2004 (received for review July 6, 2004)

Various physical, social, and biological systems generate complex fluctuations with correlations across multiple time scales. In physiologic systems, these long-range correlations are altered with disease and aging. Such correlated fluctuations in living systems have been attributed to the interaction of multiple control systems; however, the mechanisms underlying this behavior remain unknown. Here, we show that a number of distinct classes of dynamical behaviors, including correlated fluctuations characterized by  $1/f$  scaling of their power spectra, can emerge in networks of simple signaling units. We found that, under general conditions, complex dynamics can be generated by systems fulfilling the following two requirements, (i) a “small-world” topology and (ii) the presence of noise. Our findings support two notable conclusions. First, complex physiologic-like signals can be modeled with a minimal set of components; and second, systems fulfilling conditions i and ii are robust to some degree of degradation (i.e., they will still be able to generate  $1/f$  dynamics).

Complex systems are typically composed of interacting units that communicate information and are able to process and withstand a broad range of stresses (1–4). In physiology, free-running healthy systems typically generate complex output signals that have long-range correlations [i.e., a  $1/f$  decay of the power spectra for low frequencies (\*\*, 5–7)]. Deviations from the  $1/f$  pattern have been associated with disease or aging in various contexts (3, 8).

Despite its practical and fundamental interest (9), the origin of such correlated dynamics remains an unsolved problem (4). Until recently, attention has focused primarily on the complexity of the specific physiologic subsystems or on the nature of the nonlinear interactions between them (10–12). In particular, Boolean variables (which can take one of two values, 0 or 1) and Boolean functions have been extensively used to model the state and dynamics of complex systems (see ref. 12 for an introduction). The reason such a “simplistic” description may be appropriate arises from the fact that Boolean variables provide good approximations to the nonlinear functions encountered in many control systems (10, 13–15). Random Boolean networks (RBNs) were proposed by Kauffman (10) as models of genetic regulatory networks, and they have also been studied in other contexts (13, 14). Wolfram (15), in contrast, proposed that cellular automata models, which are a class of ordered Boolean networks with identical units, may explain the real-world complexity. Neither of these two classes of models has been shown to generate the complex dynamics with  $1/f$  fluctuations observed in healthy physiologic systems.

Here, we propose a modeling approach (Fig. 1*a*) that departs from traditional approaches in that we pay special attention to the topology of the network of interactions (4) and the role of noise (16). Our model is rooted in the following two considerations that are observed frequently in real-world systems. (i) The units in the system are connected mostly locally but also with some long-range connections, giving rise to so-called “small-world” topology (17, 18); and (ii) the interaction between the units is affected by “noisy” communication and/or by noisy

stimuli (19–23). We demonstrate that simple rules, such as the majority rule, are able to generate signal with complex fluctuations under simple, but physiologically relevant, conditions.

## Methods

**The Model.** We placed the Boolean units comprising the network on the nodes of a one-dimensional ring and established bidirectional nearest-neighbor connections (Fig. 1). Then, we added  $k_e N$  additional unidirectional links (where  $k_e$  is the mean excess connectivity and  $N$  is the number of units in the system) between pairs of randomly selected units. Hence, each unit had a set of links through which incoming signals arrive and that the unit then processes. During the system evolution, the state of each neighbor was replaced by a random value with probability  $\eta$ , which parameterizes the intensity of the noise.

We assigned to each unit  $i = 1, \dots, N$  a Boolean function  $T_i$ , which determined the way that the states of the neighbors and its own state were processed. We restricted our study to Boolean functions that had only the following two “effective” inputs: the state of the unit and the average state of all other neighbors. This restriction yielded 64 unique symmetric Boolean functions (see Figs. 2 and 7–11 and *Supporting Methods*, which are published as supporting information on the PNAS web site), and it had the advantage of permitting a topology-independent implementation of the Boolean functions, thus enabling a systematic study of the effect of different rules on the dynamics of the system.

**Quantification of the Dynamical Behavior of the System.** We started all of our numerical simulations with a random initial configuration and let the system evolve synchronously according to the rules of the model. We defined the state  $\mathcal{S}(t)$  of the system as the sum of the states  $\sigma_i$  of all of the Boolean units as follows:

$$\mathcal{S}(t) = \sum_i \sigma_i(t). \quad [1]$$

We recorded  $\mathcal{S}(t)$  during the course of the simulation (see Fig. 3*a–c*) and quantified the complexity of the time series generated in terms of its autocorrelation function (3). We applied the detrended fluctuation-analysis method (5), which quantifies long-range time correlations in the dynamical output of a system by means of a single scaling exponent  $\alpha$  (Fig. 3*d*). Brownian noise

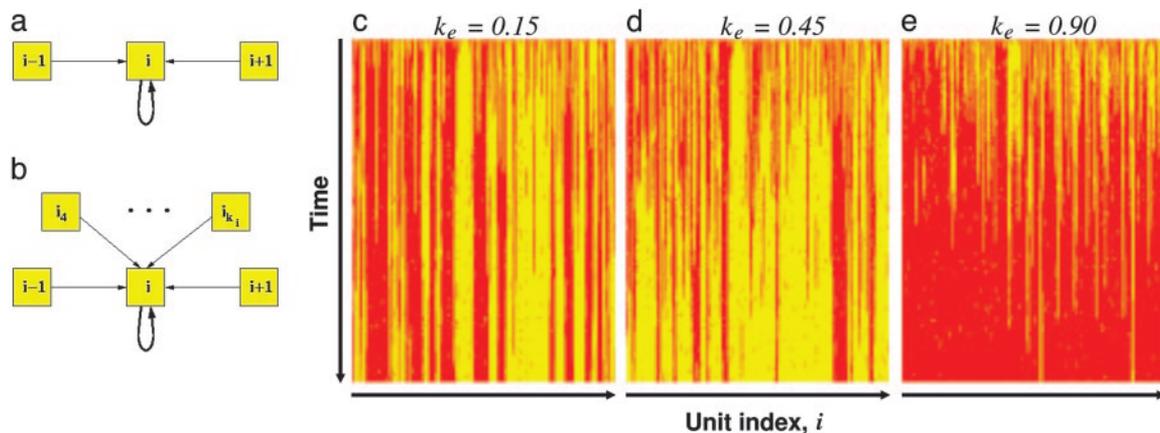
This paper was submitted directly (Track II) to the PNAS office.

Abbreviation: RBN, random Boolean network.

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\*\*A power-law-decaying power spectrum  $\mathcal{S}(f) \propto f^{-\beta}$  is the signature of a signal with power-law-decaying correlations. The case  $\beta = 2$  corresponds to a Brownian noise, whereas  $\beta = 0$  corresponds to a completely uncorrelated “white” noise. The intermediate case,  $\mathcal{S}(f) \propto 1/f$ , is a “compromise” between the small-time-scale roughness but large-time-scale smoothness of white noise and the small-time-scale smoothness but large-time-scale roughness of Brownian noise.

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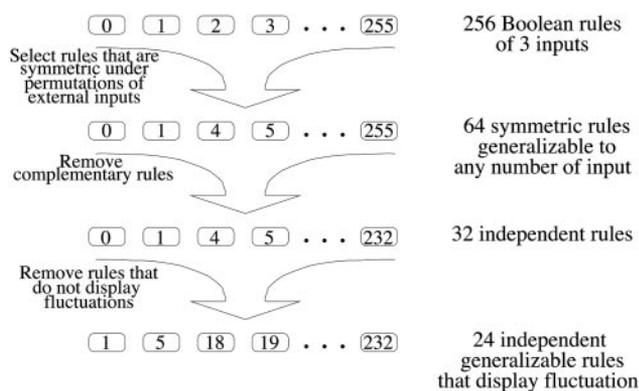


**Fig. 1.** Emergence of complex dynamics in simple signaling networks. (a) The units constituting the network, which are located on the nodes of a one-dimensional lattice, have bidirectional nearest-neighbor connections. (b) A number  $k_e N$  of additional unidirectional links is established between pairs of randomly selected units, where  $k_e$  is the mean excess connectivity and  $N$  is the number of units in the system. At time  $t = 0$ , we assign to each unit  $i = 1, \dots, N$  a state  $\sigma_i(0)$  randomly chosen from the set  $\{0, 1\}$  and a Boolean function  $T_i$  (Eq. 1). This Boolean function (or rule) determines the way in which the inputs are processed. Each unit effectively processes two inputs, one unit corresponding to the average state of its neighbors and one unit corresponding to its own state. With probability  $\eta$ , a unit “reads” a random Boolean variable instead of the state of a neighbor, where the parameter  $\eta$  quantifies the intensity of the noise. Note that the noise does not alter the state of the units but only the value read by its neighbor. At each subsequent time step, each unit updates its state synchronously according to its Boolean function. (c–e) Time evolution of systems comprising 512 units with  $T_i = 232$  for all units,  $\eta = 0.1$ , and  $k_e = 0.15$  (c),  $k_e = 0.45$  (d), and  $k_e = 0.90$  (e). Red indicates  $\sigma_i(t) = 1$ , and yellow indicates  $\sigma_i(t) = 0$ . The time evolution for systems starting from the same initial configuration and using the same sequence of random numbers is shown. Thus, the difference in the dynamics is uniquely due to the different number of long-distance links. For  $k_e = 0.15$ , the system quickly evolves toward a configuration with several clusters in which all of the units are in the same state. The boundaries of these clusters drift because of the noise, but the state of the system  $\mathcal{S}(t)$  is quite stable, and the dynamics are close to Brownian noise. In contrast, for  $k_e = 0.90$ , a large stable cluster develops and the state of the system changes only when some units change state because of the effect of the noise. This process yields white-noise dynamics. For  $k_e = 0.45$ , clusters are formed, but they are no longer stable, in contrast to what happens for small  $k_e$ . In this case, information propagates through the random links, which can lead to a change in the state of one or more units inside a cluster. Our results suggest that because these long-range connections exist on all length scales, they lead to long-range correlations in the dynamics and the observed  $1/f$  behavior (Fig. 3b).

corresponds to  $\alpha = 1.5$ , whereas uncorrelated white noise corresponds to  $\alpha = 0.5$ . For many physiologic signals, one observes  $\alpha \approx 1$ , corresponding to  $1/f$  behavior, which can be seen as a “trade off” between the two previous cases (3).

## Results

**RBNs.** The RBN model corresponds to a completely random network with randomly selected Boolean rules for the units. As



**Fig. 2.** Selection of Boolean rules for investigation. Our goal is to investigate Boolean functions that display nontrivial dynamics and can be generalized to any number of inputs. To this end, we start from 256 rules of three inputs but then restrict our attention to the ones that are symmetric under permutations of the external inputs. This selection results in 64 Boolean rules. However, each rule has another rule that is its complement (i.e., that displays the same dynamics when switching zeros and ones) or inverse (i.e., that displays the same dynamics when taken every other step). Because these pairs of rules have equivalent dynamics, we need to investigate only 32 independent rules. Of these rules, eight do not display fluctuations, even in the presence of noise, resulting in 24 independent rules that could present complex fluctuations. The phase spaces of each of these 24 rules are shown in Figs. 7–11.

shown in Fig. 4a, we find white-noise dynamics for essentially any pair of values of  $k_e$  and  $\eta$  within the ranges considered, suggesting that, even in the presence of noise, a system of random Boolean functions cannot generate  $1/f$  dynamics. This result is not unexpected because the random collection of Boolean functions comprising the system prevents the development of any order or predictability in the dynamics.

**Cellular-Automata Models with Small-World Topology and Noise.** We systematically study the 64 symmetric rules (see Figs. 2 and 7–11 and *Supporting Methods*) for different pairs of values of  $k_e$  and  $\eta$ . Some of the rules have parallels to physiologically meaningful dynamics. Rule 232 is a majority rule; that is, each unit will be active at the next time step only if the majority of its neighbors is active presently. Rule 50 is a threshold rule with refractory time period; that is, whenever the inputs of the neighbors surpass a certain value, a unit becomes active in the next time step and then will be inactive for at least one time step. The 64 symmetric rules lead to three qualitatively distinct phase spaces (Fig. 4b–d). Rule 232, the majority rule, which is representative of the first type of phase space, displays three distinct types of dynamical behavior (Fig. 4b). For small  $k_e$ , we find mostly Brownian-like scaling. For large  $k_e$ , we find mostly white-noise dynamics. Of greatest interest, for intermediate values of  $k_e$  and for a broad range of values of the intensity of the noise  $\eta$ , we find  $1/f$  fluctuations.

Rule 50, which is representative of a second type of phase space, displays fewer types of dynamical behavior. In particular, we find only a narrow range of noise intensities (with a weak dependence on  $k_e$ ) for which the dynamics display  $1/f$  correlations. For  $\eta \approx 0.1$ , the dynamics become uncorrelated (Fig. 4c). Rule 160, which is representative the third type of phase space, displays white-noise dynamics for all values of  $k_e$  and  $\eta$  (Fig. 4d).

Note that for  $k_e = 0$  (i.e., when the network is a one-





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